Mid term test

Total Marks : 35

State true or false and explain. (Each question carries 2 marks)

1. The distributional derivative of $f(x) = \log|x|$ is $\frac{1}{x}$.
2. The function $e^{|x|}$ defines a tempered distribution in $I\!R$.
3. Let f_n, f belong to $L^1_{loc}(\Omega)$ and $f_n \to f$ a.e in Ω. Then $f_n \to f$ in $\mathcal{D}'(\Omega)$.
4. There exists an f in $L^1(I\!R^n)$ whose Fourier transform is $sin\ x$.
5. For every distribution T, sing. supp T $\subset$ supp T.
6. If u and u^2 are harmonic, then u is a constant.

Answer the following questions.

1. Let u be a harmonic function in Ω whose normal derivative $\frac{\partial u}{\partial \nu}$ vanishes on $\partial\Omega$. Show that u is constant in each connected component of Ω. [3]
2. Let L be a linear differential operator with constant coefficients which is hypoelliptic. Show that every fundamental solution for L is smooth in $I\!R^n \setminus \{0\}$. [3]
3. (a) Define the space $\mathcal{E}(\Omega)$ and $\mathcal{E}'(\Omega)$.
   (b) Show that if $T \in \mathcal{E}'(\Omega)$, then the support of T is compact.
   (c) Let $T = \Sigma_{n=0}^{\infty} D^n \delta_{\frac{1}{n}}$. Show that $T \in \mathcal{D}(0,1)$ but $T \notin \mathcal{E}(0,1)$ [6]
4. Let $\{u_n\}$ be a monotone increasing sequence of harmonic functions in a domain Ω and suppose that for some point $y \in \Omega$ the sequence $\{u_n(y)\}$ is bounded. Then the sequence converges uniformly on any bounded subdomain $\Omega_0 \subset \Omega$ to a harmonic function. [4]
5. Find the fundamental solution of Laplace operator in $\mathbb{R}^n$, for $n \geq 2$. [4]
6. Any bounded sequence of harmonic functions on a domain Ω contains a subsequence converging uniformly on compact subdomains of Ω to a harmonic function. [3]